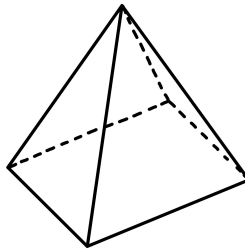**Math Adventures**
**Week 1: 3D Shapes**

A **polyhedron** is a 3D shape made up of vertices, straight edges, and flat polygonal faces.
- A **vertex** is a corner of a geometric shape.
- An **edge** is a line segment that connects two vertices.
- A **face** is a flat surface that forms part of a 3D shape.

1. Answer the questions below about the following polyhedron.

    a. How many vertices are there?
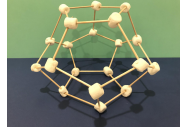
    b. How many edges are there?

    c. How many faces are there?

2. Answer the following questions about polyhedra.
    a. Can a polyhedron have 3 vertices? Why or why not?

    b. What is the smallest number of vertices a polyhedron can have?

    c. What is the smallest number of edges a polyhedron can have?

    d. What is the smallest number of faces a polyhedron can have?

A **regular polygon** is a 2D shape where all the edges are equal and all the angles are equal.

A **Platonic solid** is a type of polyhedron where all the faces are identical, regular polygons.

3.   Build the five Platonic solids with marshmallows and toothpicks. Fill out the table below.

|  | Tetrahedron | Cube | Octahedron | Icosahedron | Dodecahedron |
|---|---|---|---|---|---|
| Faces (*F*) |  |  |  |  |  |
| Vertices (*V*) |  |  |  |  |  |
| Edges (*E*) |  |  |  |  |  |
| $F + V - E =$ |  |  |  |  |  |

A **pyramid** is a type of polyhedron where the base is a polygon and all the vertices of the base are connected to a special vertex called the **apex**.

4.   Fill out the table below about pyramids with different polygon bases.

|  | Bases | | | | |
|---|---|---|---|---|---|
|  | Triangle (3-gon) | Quadrilateral (4-gon) | Pentagon (5-gon) | Hexagon (6-gon) | *n*-gon |
| Faces (*F*) |  |  |  |  |  |
| Vertices (*V*) |  |  |  |  |  |
| Edges (*E*) |  |  |  |  |  |
| $F + V - E =$ |  |  |  |  |  |

5.   Looking back at the last row of each table, we can develop a formula that is true for all polyhedra. It is called Euler's formula.

   **Euler's formula:** $F + V - E =$

6. Answer the following questions about pyramids.

    a. If a pyramid has 10 vertices, how many edges does it have? How many faces does it have?

    b. If a pyramid has 20 edges, how many vertices does it have? How many faces does it have?

    c. If a pyramid has $F$ faces, how many edges does it have?

    d. Is it possible for a pyramid to have 2019 vertices?

    e. Is it possible for a pyramid to have 2019 edges?

## Lesson Summary

A **polygon** is a 2D shape with straight edges.
- A **vertex** is a corner where 2 edges meet.
- An **edge** is a line segment that connects 2 vertices.

A **regular polygon** is a 2D shape where all the edges are equal and all the angles are equal.

A **polyhedron** is a 3D shape made up of vertices, straight edges, and flat polygonal faces.
- A **face** is a flat surface that forms part of a 3D shape.
- Polyhedra have no curved surfaces.

A **Platonic solid** is a type of polyhedron where all the faces are identical, regular polygons. There are five Platonic solids:

Tetrahedron    Cube    Octahedron    Icosahedron    Dodecahedron

You can build them out of toothpicks and marshmallows!

A **pyramid** is a type of polyhedron where the base is a polygon and all the vertices of the base are connected to a special vertex called the **apex**.

For every polyhedron, $F + V - E = 2$, where $F$ represents the number of faces, $V$ represents the number of vertices, and $E$ represents the number of edges. This formula is called **Euler's formula**.

The first step to mathematical discovery is recognizing patterns! Today, we recognized patterns in the numbers of faces, vertices, and edges in various polyhedra and came up with Euler's formula. Later, when we acquire enough skills, we can figure out how to prove that Euler's formula is true for all polyhedra.

We also noticed that for a pyramid that has a base with $n$ number of sides, we can calculate the numbers of faces, vertices, and edges it has using the following formulas:

|  | $n$-gon base |
| --- | --- |
| Faces ($F$) | $n + 1$ |
| Vertices ($V$) | $n + 1$ |
| Edges ($E$) | $2n$ |

References: Olga Radko Endowed Math Circle archive