Name: ________________________

Math Adventures
Week 4: The Fibonacci Sequence and the Golden Ratio

In the 1200s, an Italian mathematician named Leonardo Fibonacci asked the question, "If we start with 2 newborn rabbits in a pen, how many rabbits will there be after 1 year?"

Rabbit Rules:

- Start with 1 pair of newborn rabbits.
- When the rabbits are 1 month old, they are teenagers. Teenagers cannot have children.
- When the rabbits are 2 months old, they are adults. A pair of rabbits will give birth to 1 pair of newborn rabbits the same month they become adults.
- A pair of adult rabbits gives birth to 1 pair of newborn rabbits each month.
- Rabbits never die. Once a rabbit becomes an adult, it remains an adult forever.

1. Suppose there are 5 pairs of newborn rabbits in a given month.
   a. How many pairs of adults must there be in that same month?

   b. How many pairs of teenagers will there be in the next month?

2. Suppose there is 1 pair of newborn rabbits in December (Month 0).
   a. How many pairs of each type of rabbit will there be in January (Month 1)?
      i. Newborn:
      ii. Teenager:
      iii. Adult:

   b. What about February (Month 2)?
      i. Newborn:
      ii. Teenager:
      iii. Adult:

3. Starting with 1 pair of newborn rabbits born in Month 0, fill out the chart below.

	Month												
	0	1	2	3	4	5	6	7	8	9	10	11	12
Newborn	1												
Teenager	0												
Adult	0												
Total	1												

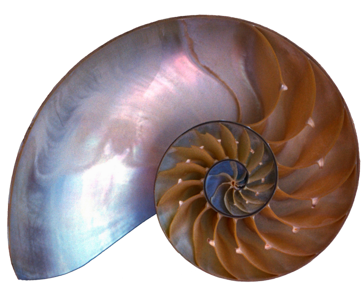
The sequence of numbers in the bottom row that represents the total number of pairs of rabbits is called the **Fibonacci sequence**!

4. How is the total number of pairs of rabbits in Month 2 related to the total number of pairs in Month 0 and Month 1?

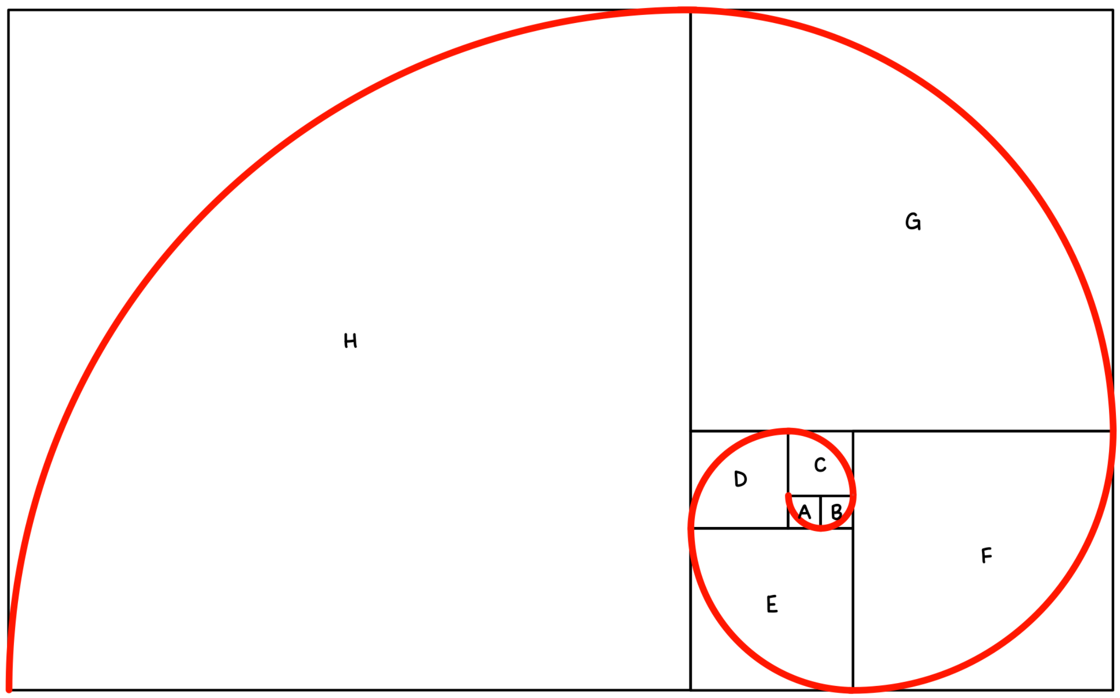
5. How is the total number of pairs in Month 5 related to the total number of pairs in Month 3 and Month 4?

6. How is the total number of pairs in any month related to the total number of pairs in the previous months?

The Fibonacci sequence is also seen in other instances of nature! For example, the spiral of the nautilus seashell is related to the Fibonacci sequence.



7. In the diagram below, assume that the side lengths of the smallest, inside squares (A and B) are 1 cm long. Find the side lengths of the rest of the squares by starting from the center and working outward. Fill out the table below with the side lengths.



A	B	C	D	E	F	G	H
1	1						

8. What do you notice about the side lengths?

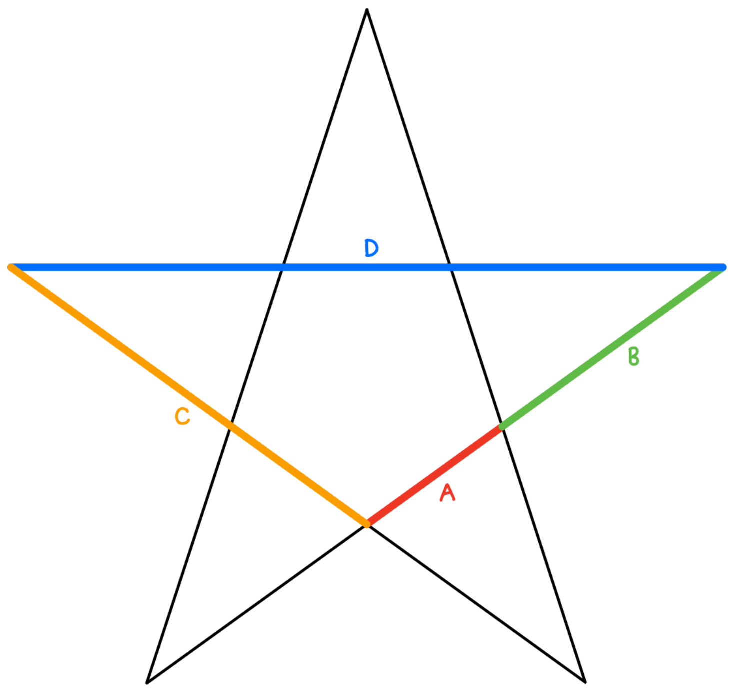
9. Calculate the ratios of successive terms in the Fibonacci sequence using the table above. Round to the nearest hundredth.

$\frac{B}{A} =$ _________ $\frac{C}{B} =$ _________ $\frac{D}{C} =$ _________ $\frac{E}{D} =$ _________

$\frac{F}{E} =$ _________ $\frac{G}{F} =$ _________ $\frac{H}{G} =$ _________

10. What number do your ratios seem to approach?

11. In the pentagram (5-pointed star) below, measure the lengths of A, B, C, and D in centimeters to the nearest tenth of a centimeter.



A: ________ cm   B: ________ cm   C: ________ cm   D: ________ cm

12. Calculate the following ratios using the lengths you found above. Round to the nearest tenth. Then, find the average of your three ratios.

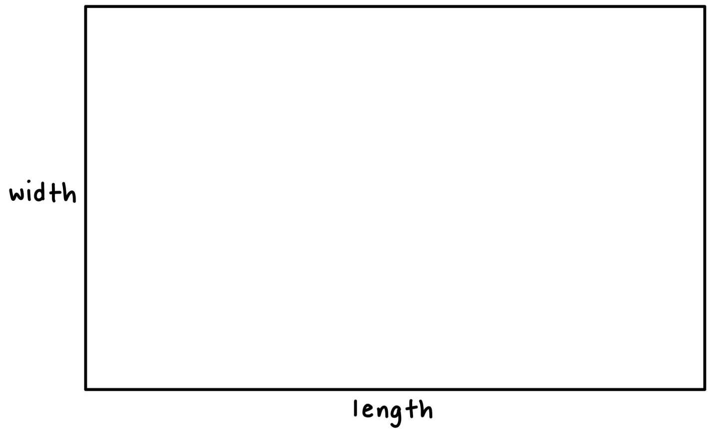
$\frac{B}{A}$ = ________ $\frac{C}{B}$ = ________ $\frac{D}{C}$ = ________   Average = ________

The ratios found in the nautilus, the pentagram, and many other shapes and patterns in nature are all a number called **phi**. The ratios of successive terms in the Fibonacci sequence approach phi as well. Phi is represented by the Greek symbol **Φ** and is also known as the **golden ratio**.

Rounded to the nearest thousandth, Φ = ________.

13. Draw the most beautifully proportioned rectangle you can in the space below.

The Ancient Greeks thought the following rectangle was the most beautiful rectangle of all, and used it frequently in their architecture. What makes it so special? Let's find out!



14. Measure the length and width of the rectangle to the nearest tenth of a centimeter.

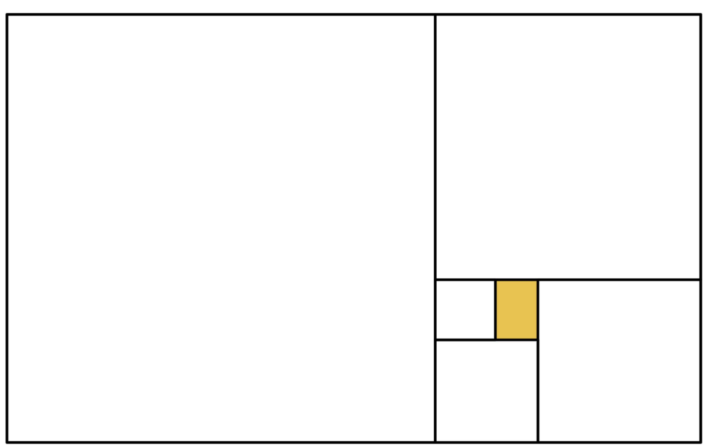
    Length: ________ cm

    Width: ________ cm

15. Calculate the ratio of the length to the width. Round to the nearest tenth.

    $\frac{length}{width}$ = ________

The **golden rectangle** is a rectangle whose sides are in the golden ratio ($\frac{length}{width} = \Phi$). If we cut a square from one side of a golden rectangle, we get another golden rectangle.



Lesson Summary

The Fibonacci sequence was developed by Leonardo Fibonacci in trying to answer the question, “If we start with 2 newborn rabbits in a pen, how many rabbits will there be after 1 year?”

- There will be 233 rabbits after 1 year.

Each term in the Fibonacci sequence is the **sum of the previous two terms**.

The Fibonacci sequence is also seen in other instances in nature:

- The lengths in the spiral of the nautilus seashell
- Plant growth
    - Seeds
    - Flower petals
    - Pinecones

The ratios found in the nautilus, the pentagram, and many other shapes and patterns in nature are all a number called **phi**. The ratios of successive terms in the Fibonacci sequence approach phi as well. Phi is represented by the Greek symbol **Φ** and is also known as the **golden ratio**.

Rounded to the nearest thousandth, $\Phi = 1.618$.

The **golden rectangle** is a rectangle whose sides are in the golden ratio ($\frac{length}{width} = \Phi$). The Ancient Greeks thought the golden rectangle was the most beautiful rectangle of all, and used it frequently in their architecture. If we cut a square from one side of a golden rectangle, we get another golden rectangle.

References: Olga Radko Endowed Math Circle archive, TeachEngineering